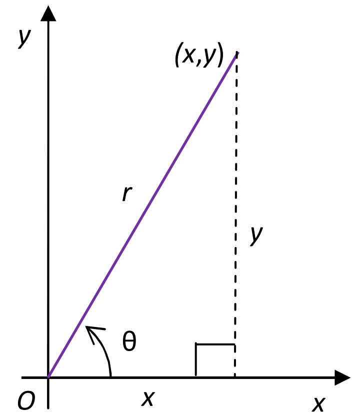
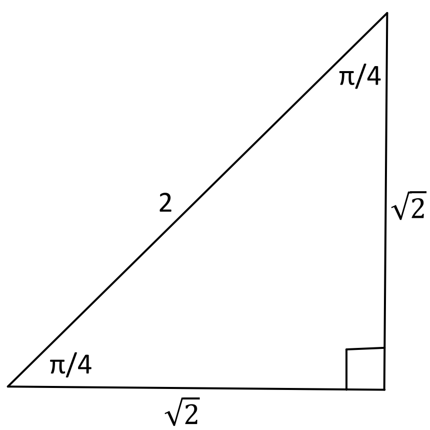
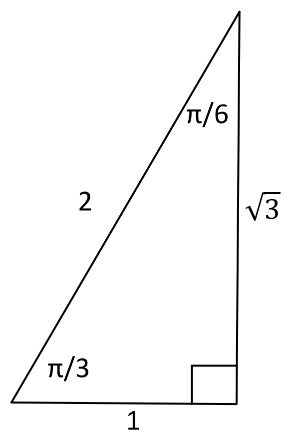
Right-Triangle Trigonometry

$\sin\theta = \frac{y}{r} = \frac{\text{side opposite } \theta}{\text{hypotenuse}}$	$\csc\theta = \frac{1}{\sin\theta}$	
$\cos\theta = \frac{x}{r} = \frac{\text{side adjacent } \theta}{\text{hypotenuse}}$	$\sec\theta = \frac{1}{\cos\theta}$	
$\tan\theta = \frac{y}{x} = \frac{\text{side opposite } \theta}{\text{side adjacent } \theta}$	$\cot\theta = \frac{1}{\tan\theta}$	

π radians $= 180°$

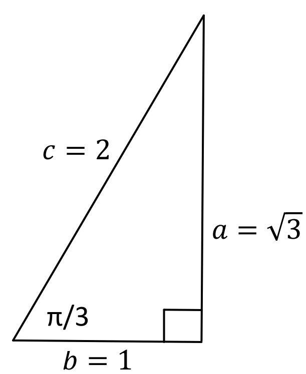
Two Special Triangles:	
$45° - 45° - 90°$	$30° - 60° - 90°$
	

Example Evaluate and simplify $\sec^2\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right)$

Solution:

$$\sec^2\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) = \left(\frac{1}{\cos(\pi/3)}\right)^2 \tan\left(\frac{\pi}{3}\right)$$

$$= \left(\frac{1}{1/2}\right)^2 \left(\frac{\sqrt{3}}{1}\right) = 2^2 \times \frac{\sqrt{3}}{1} = \mathbf{4\sqrt{3}}$$



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Example Evaluate and simplify $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}\cos 60° & 0 & \sin 60°\\0 & 1 & 0\\-\sin 60° & 0 & \cos 60°\end{bmatrix}$.

Solution:

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}\cos 60° & 0 & \sin 60°\\0 & 1 & 0\\-\sin 60° & 0 & \cos 60°\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}1/2 & 0 & \sqrt{3}/2\\0 & 1 & 0\\-\sqrt{3}/2 & 0 & 1/2\end{bmatrix}$$

$$= \begin{bmatrix}(1)(1/2)+(0)(0)+(0)(-\sqrt{3}/2) & (1)(0)+(0)(1)+(0)(0) & (1)(\sqrt{3}/2)+(0)(0)+(0)(1/2)\\(0)(1/2)+(1)(0)+(0)(-\sqrt{3}/2) & (0)(0)+(1)(1)+(0)(0) & (0)(\sqrt{3}/2)+(1)(0)+(0)(1/2)\\(0)(1/2)+(0)(0)+(0)(-\sqrt{3}/2) & (0)(0)+(0)(1)+(0)(0) & (0)(\sqrt{3}/2)+(0)(0)+(0)(1/2)\end{bmatrix}$$

$$= \begin{bmatrix}\mathbf{1/2} & \mathbf{0} & \mathbf{\sqrt{3}/2}\\\mathbf{0} & \mathbf{1} & \mathbf{0}\\\mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

Example Given $0 \le \theta < \frac{\pi}{2}$ with $\tan\theta = 2$, evaluate and simplify $\ln(\sec\theta + \tan\theta)$.

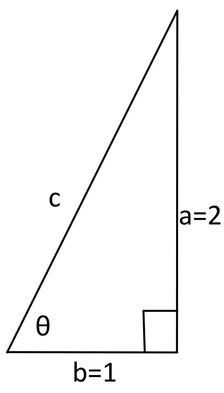
Solution:

(Use the definition $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ to draw the right triangle involving θ)

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{2}{1}$$

(Use Pythagorean Theorem to calculate the hypotenuse)

$$c^2 = a^2 + b^2 \Rightarrow c^2 = 2^2 + 1^2 \Rightarrow c^2 = 5 \Rightarrow c = \sqrt{5}$$



(Use the definitions $\sec\theta = \frac{1}{\cos\theta}$ and $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ to evaluate and simplify the given expression)

$$\ln(\sec\theta + \tan\theta) = \ln\left(\frac{1}{1/\sqrt{5}} + \frac{2}{1}\right) = \mathbf{\ln(\sqrt{5}+2)}$$