Linear Equations and Inequalities in 2 variables

Graphing Lines in the Rectangular Coordinate System

- Sketching lines by intercepts
- Sketching vertical lines

Slope of a Line

- $\text{Slope}: m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{change in y - coordinate}}{\text{change in x - coordinate}} = \dfrac{\text{rise}}{\text{run}}$
- Slope of horizontal lines are zero; slopes of vertical lines are undefined
- Slope of parallel lines: $m_1 = m_2$
- $\text{Slope of perpendicular lines}: m_1 = -\dfrac{1}{m_2}$

Three Forms for the Equation of a Line

- Point-slope form: $y - y_1 = m(x - x_1)$
- Slope-intercept form: $y = mx + b$
- Standard Form: $Ax + By = C$

Example Find an equation for the line passing through $\left(\frac{\sqrt{3}}{2}, 0\right)$ with slope 3.

Solution:

(Apply the point-slope form) $y - 0 = 3\left(x - \frac{\sqrt{3}}{2}\right)$

(Expand) $y - 0 = 3x - \frac{3\sqrt{3}}{2}$

(Simplify by collecting like terms, write answer in a particular form if so desired)

$\boldsymbol{y = 3x - \frac{3\sqrt{3}}{2}}$

Exercise Find an equation for the line

- passing through $\left(4\sqrt{2}, 0\right)$ with slope $\frac{1}{5}$ [Answer: $y = \frac{1}{5}x - \frac{4\sqrt{2}}{5}$]
- containing the points $(1, -4)$ and $(2,3)$ [Answer: $y = 7x - 11$]

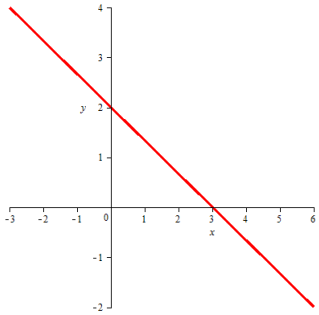
Linear Equations and Inequalities in 2 variables

Example Sketch the graph of the function $y = 2 - \frac{2}{3}x$.

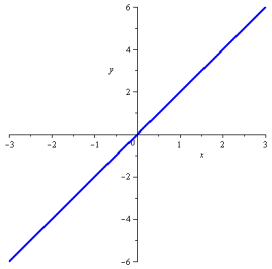
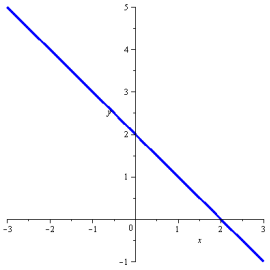
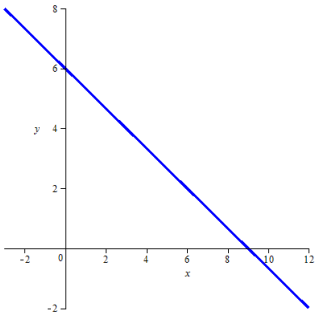
Solution:

(The function fits the slope-intercept form $y = mx + b$ of a line; identify the slope m and the y-intercept b and sketch the graph accordingly; recall that slope $= \frac{\text{rise}}{\text{run}}$)

$y = 2 - \frac{2}{3}x = \left(-\frac{2}{3}\right)x + 2 \Longrightarrow$ slope $m = -\frac{2}{3}, y -$ intercept $= 2$



Exercise Sketch the graph of the function

$y = 2x$ Answer:  (Line with slope $m = 2$ and y-intercept $b = 0$)	$y = 2 - x$ Answer:  (Line with slope $m = -1$ and y-intercept $b = 2$)
$2x + 3y = 18$ Answer: $2x + 3y = 18 \Leftrightarrow y = -\frac{2}{3}x + 6$  (Line with slope $m = -\frac{2}{3}$ and y-intercept $b = 6$)	

Linear Equations and Inequalities in 2 variables

(System of) Linear Inequalities

- Graphing linear inequality using the Test Point Method

Example Sketch the solution set of the inequality $x + y > 0$.

Solution:

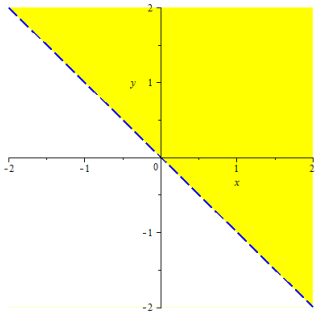
(Replace the inequality sign in the inequality with an equality sign, sketch the graph of the equation)

$x + y = 0 \Longrightarrow y = -x$ (a line passing through the origin with slope –1)

(This graph divide the *xy*-coordinate planes into several regions, and we determine which region we should include in the solution, that is, satisfy the inequality we try to solve, by using test points)

Region	Above $y = -x$	Below $y = -x$
Test Point (one possibility)	$(0, 1)$	$(0, -1)$
Inequality $\frac{1}{4(2+x)(5-x)} > 0$ Satisfied?	$0 + 1 > 0$?	$0 + (-1) > 0$?
Part of the Solution?	Yes	No

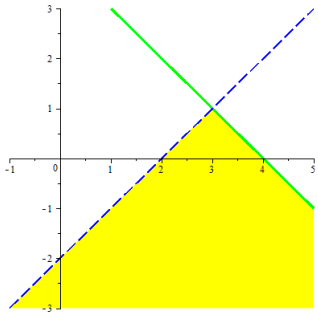
(The points on the graph satisfies the equation $x + y = 0$ and do not form part of the solution, represented by the dashed boundary line)



Linear Equations and Inequalities in 2 variables

- Graphing Compound Inequality with and/or

Exercise Sketch the solution set of the inequalities

$x + y \leq 4,\ x - y > 2$ Answer: 	$3x - y \leq 6,\ y - 3 < 0$ Answer: