Linear Equations and Inequalities (1 variable)

<u>3 types of equations</u>

- Identity: the equation is satisfied for all values of the variable

- Conditional Equation: the equation is satisfied by at least one (but not all) values of the variable

- Inconsistent Equation: the equation is satisfied by no value of the variable

<u>Example</u> Solve $7(3x + 9) = 10 - (x + 2)$.

Solution:

(Expand both sides of the equation) $21x + 63 = 10 - x - 2$

(Collect like terms and solve for x)

$$21x + x = 10 - 2 - 63 \Rightarrow 22x = -55 \Rightarrow x = -\frac{55}{22} = -\frac{5}{2}$$

(Check the answer found satisfies the original equation)

$$\text{L. H. S.} = 7\left[3\left(-\frac{5}{2}\right) + 9\right] = 7\left[-\frac{15}{2} + 9\right] = 7\left[\frac{3}{2}\right] = \frac{21}{2}$$

$$\text{R. H. S.} = 10 - \left(-\frac{5}{2} + 2\right) = 10 - \left(-\frac{1}{2}\right) = \frac{21}{2}$$

Hence $x = -\frac{5}{2}$

<u>Example</u> Solve the formula $p = np + q$ for p.

Solution:

(Collect the terms involving the variable p on the same side of the equation)

$$p - np = q$$

(Solve for p) $p(1 - n) = q \Rightarrow p = \frac{q}{1-n}$

<u>Example</u> Solve $|4x - 3| + 1 = 7$.

Solution:

$$|4x - 3| = 7 - 1 \Rightarrow |4x - 3| = 6 \Rightarrow \begin{cases} 4x - 3 = 6 \\ \text{or} \\ 4x - 3 = -6 \end{cases} \Rightarrow \begin{cases} 4x = 9 \\ \text{or} \\ 4x = -3 \end{cases} \Rightarrow \begin{cases} x = 9/4 \\ \text{or} \\ x = -3/4 \end{cases}$$

(Check the answers in the original equation)

$$x = \frac{9}{4} \Rightarrow \begin{cases} \text{L. H. S.} = \left|4\left(\frac{9}{4}\right) - 3\right| + 1 = |9 - 3| + 1 = |6| + 1 = 6 + 1 = 7 \\ \text{R. H. S.} = 7 \end{cases}$$

Linear Equations and Inequalities (1 variable)

$$x = -\frac{3}{4} \Rightarrow \begin{cases} \text{L. H. S.} = \left|4\left(-\frac{3}{4}\right) - 3\right| + 1 = |-3 - 3| + 1 = |-6| + 1 = 6 + 1 = 7 \\ \text{R. H. S.} = 7 \end{cases}$$

Hence $x = \frac{9}{4}$ **or** $x = -\frac{3}{4}$

Exercise

- Solve $6x - 7 = 2$ [Answer: $\frac{3}{2}$]

- Solve $2(5 - 3x) = 8 - 3(x + 2)$. [Answer: $\frac{8}{3}$]

- Solve the formula $P = 2L + 2W$ for L. [Answer: $L = \frac{P - 2W}{2}$ or $L = \frac{P}{2} - W$]

- Solve the formula $A = P + Prt$ for P. [Answer: $P = \frac{A}{1 + rt}$]

- Solve $|x - 3| = 2$. [Answer: $x = 1, 5$]

Addition/Multiplication Property of Equality in one variable

- $a > b \Rightarrow a + c > b + c$ and $a - c > b - c$ (similar results hold for ≥)

- $a > b \Rightarrow \begin{cases} \text{for any } c > 0: \quad ac > bc \quad \text{and} \quad \frac{a}{c} > \frac{b}{c} \\ \text{for any } c < 0: \quad ac < bc \quad \text{and} \quad \frac{a}{c} < \frac{b}{c} \end{cases}$

Exercise Solve the inequality

- $3x - 5 < 6 - 2x$ [Answer: $x < \frac{11}{5}$ or $\left(-\infty, \frac{11}{5}\right)$]

- $13 - 7x \geq 10x - 4$ [Answer: $x \leq 1$ or $(-\infty, 1]$]

Solving compound inequality using the connective "and"/"or"

Example Solve $\frac{2}{3} \leq -\frac{4}{5}(x - 3) < 1$.

Solution:

$$\frac{2}{3}\left(-\frac{5}{4}\right) \geq x - 3 > 1\left(-\frac{5}{4}\right) \quad \Rightarrow \quad -\frac{5}{6} \geq x - 3 > -\frac{5}{4} \quad \Rightarrow \quad -\frac{5}{6} + 3 \geq x > -\frac{5}{4} + 3$$

$$\Rightarrow \quad \frac{13}{6} \geq x > \frac{7}{4}$$

Example Solve $3x + 7 \leq 2$ or $2x + 3 \geq 5$.

Solution:

$$3x + 7 \leq 2 \text{ or } 2x + 3 \geq 5 \quad \Rightarrow \quad 3x \leq -5 \text{ or } 2x \geq 2 \quad \Rightarrow \quad x \leq -\frac{5}{3} \text{ or } x \geq 1$$

Linear Equations and Inequalities (1 variable)

Exercise Solve the inequality

- $-3 < 2x + 5 \leq 7$ [Answer: $-4 < x \leq 1$ or $(-4,1]$]
- $2x - 5 \leq -7$ or $2x - 5 > 1$ [Answer: $x \leq -1$ or $x > 3$, or $(-\infty, -1] \cup (3, \infty)$]

Example Solve $\left|\frac{x-3}{7}\right| < 1$.

Solution:

(Rewrite the inequality without using the absolute value notation) $-1 < \frac{x-3}{7} < 1$

(Solve the inequality by using the properties of inequalities) $-7 < x - 3 < 7$

$\Rightarrow -4 < x < 10,$ or using interval notation, $(-4, 10)$.

Exercise Solve the inequality

- $|3x + 2| < 5$ [Answer: $-\frac{7}{3} < x < 1$ or $\left(-\frac{7}{3}, 1\right)$]
- $|x - 3| < 1$ [Answer: $2 < x < 4$ or $(2,4)$]
- $\left|\frac{x-4}{6}\right| < \frac{1}{2}$ [Answer: $1 < x < 7$ or $(1,7)$]
- $\left|\frac{x+2}{3}\right| \leq 2$ [Answer: $-8 \leq x \leq 4$ or $[-8,4]$]
- $|5 - 2x| \geq 1$ [Answer: $x \leq 2$ or $x \geq 3$, or $(-\infty, 2] \cup [3, \infty)$]

Trichotomy Property

For any two real numbers a and b, exactly one of the three conditions is satisfied:

$$a < b, \quad \text{or} \quad a = b, \quad \text{or} \quad a > b$$

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