Integral Exponents

Integral Exponents, Scientific Notation

- Negative Exponents $(a \neq 0): a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n; \frac{1}{a^{-n}} = a^n$
- Zero Exponent: $a^0 = 1$ (for $a \neq 0$)
- Product Rule: $a^m \cdot a^n = a^{m+n}$
- Quotient Rule: $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$

Exponents: Power Rules

- Power of a Power Rule: $(a^m)^n = a^{mn}$
- Power of a Product Rule: $(ab)^n = a^n \cdot b^n$
- Power of a Quotient Rule: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example Simplify $\left(\frac{24a^{10}b^{-8}}{12ab^4}\right)^{-3}$ and write your answer using only positive exponents.

Solution:

$$\left(\frac{24a^{10}b^{-8}}{12ab^4}\right)^{-3} = \left(\frac{24}{12} \times \frac{a^{10}}{a} \times \frac{b^{-8}}{b^4}\right)^{-3} = (2a^{10-1}b^{-8-4})^{-3} = (2a^9b^{-12})^{-3}$$

$$=2^{-3}a^{(9)(-3)}b^{(-12)(-3)} = 2^{-3}a^{-27}b^{36} = \frac{b^{36}}{2^3a^{27}} = \boldsymbol{\frac{b^{36}}{8a^{27}}}$$

Example Solve the identity $\frac{L}{T^2} = L^n\left(\frac{L}{T}\right)^m$ for m and n.

Solution:

$$\frac{L}{T^2} = L^n\left(\frac{L}{T}\right)^m \Longrightarrow \frac{L}{T^2} = L^n \cdot \frac{L^m}{T^m} \Longrightarrow LT^{-2} = L^{n+m}T^{-m}$$

(Equate powers of the variables)

$$\left.\begin{cases}\text{Powers of } L: & n+m=1 \\ \text{Powers of } T: & -m=-2 \quad \Rightarrow \quad \boldsymbol{m=2}\end{cases}\right\} \Rightarrow n+2=1 \Rightarrow n=1-2 \Rightarrow \boldsymbol{n=-1}$$

Example Given n is a positive integer, evaluate and simplify $\left|\frac{(x+2)^{n+1}}{(n+1)3^{n+1}} \div \frac{(x+2)^n}{n\cdot 3^n}\right|$.

Solution:

$$\left|\frac{(x+2)^{n+1}}{(n+1)3^{n+1}} \div \frac{(x+2)^n}{n\cdot 3^n}\right| = \left|\frac{(x+2)^{n+1}}{(n+1)3^{n+1}} \times \frac{n\cdot 3^n}{(x+2)^n}\right| = \left|\frac{n}{n+1} \times \frac{3^n}{3^{n+1}} \times \frac{(x+2)^{n+1}}{(x+2)^n}\right| = \left|\frac{n}{n+1} \times \frac{1}{3} \times (x+2)\right|$$

$$= \frac{|n|\times|x+2|}{|n+1|\times|3|} = (\text{since both } n \text{ and } n+1 \text{ are positive}) = \boldsymbol{\frac{n\cdot|x+2|}{3(n+1)}}$$

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Exercise

- Simplify $\left(\frac{30x^{-3}y^2}{6xz^{-8}}\right)^{-3}$ and write your answer using only positive exponents.

  [Answer: $\frac{x^{12}}{125y^6z^{24}}$]

- Given n is a positive integer, evaluate and simplify the following
  - $\frac{n+1}{2^{n+1}} \div \frac{n}{2^n}$ [Answer: $\frac{n+1}{2n}$]
  - $\frac{4^{n+1}}{3^{n+1}+5} \div \frac{4^n}{3^n+5}$ [Answer: $\frac{4(3^n+5)}{3^{n+1}+5}$]